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The infinite component EA spin-glass model revisited

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Abstract. The infinite component limit of the Edwards–Anderson spin-glass model is studied to cubic order in the order parameter Q, below its transition temperature. A perturbation expansion around the mean-field solution is performed, and some terms in the expansion are calculated to second non-trivial order. It is found that the series presents infrared (IR) divergences below certain dimension which increases with the order of the terms; however, these divergences cancel exactly with ultraviolet (UV) divergences within the dimensional regularization scheme. In terms of this new evidence, the critical behaviour of this model, and some 'strange' results previously found are discussed.

1. Introduction

In this paper we study the low-temperature phase of the Edwards–Anderson (EA) m-component spin-glass model (Edwards and Anderson 1975) in the limit when the number of spin components m tends to infinity independently of the spatial dimensionality d. To this effect, we consider a truncated version of the Ginzburg–Landau–Wilson (GLW) free energy to cubic order $O(Q^3)$, where Q is the spin-glass parameter. This model, in this approximation, has already been studied by using different methods (Green *et al* 1982, Viana 1988 (to be refered to as LV)), and some unusual results have been found, which seem to indicate that the critical upper and lower dimensionalities of the theory coincide. On one hand, a one-loop perturbation expansion in the low temperature region fails due to the appearance of infrared (IR) divergences for $d \leqslant 8$ (LV); this is usually considered as an indication of $d_{\rm lcd} = 8$ as being the lower critical dimension of the theory, below which SG order is imposible due to fluctuations. On the other hand, by using the method of minimal substraction of ϵ poles ('t Hooft and Veltman 1872), Green *et al* (1982) have found that the upper critical dimension, above which mean-field theory provides the exact solution, is given by $d_{\rm ucd} = 8$. This last result would imply a shift in the critical behaviour of this model in the large-m limit, since it has been reported that $d_{\rm ucd} = 6$ (Green *et al* 1982) and $d_{\rm lcd} = 4$ (Bray and Moore 1979b) for the finite m (> 1) version of this model.

The explanation given to the qualitative change of behaviour of the EA m-component SG model in the $m \to \infty$ limit is the following (Bray and Moore 1979a,b, Green *et al* 1982). In 'replica language' this model contains the so-called 'quadrupole fields', which couple different spin components within the same replica (this will be shown later); for

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any finite value of m, the quadrupole fields do not play a critical role in the theory, since their 'mass' is larger than that of others by a factor of the order of $1/m$. However, for $m \to \infty$ the mass related to the quadrupole fields becomes equal to the mass of other modes. As a consequence, in this limit the quadrupole fields go 'soft' and couple different vector components, thus changing the critical behaviour.

The results previously mentioned, which seem to imply that the lower and upper critical dimensionalities of this model coincide in the $m \to \infty$ limit, are very strange. Having a theory in which the lower and upper critical dimensions are the same ($= d_c$) is not what we would expect physically. This would imply that if we slowly varied the space dimensionality from $d < d_c$ to $d > d_c$, we would go abruptly—without an intermediate region—from a regime where fluctuations are so important that order cannot exist, to another regime where fluctuations are completely irrelevant, in such a way that mean-field theory predicts exact results. In order to gain some insight into this problem, in this paper we extend the calculation of LV to include some terms of the series expansion to next non-trivial order. Subsequently, we analyse the behaviour of the series to find out whether or not the critical dimensionalities coincide and are given by $d_{lcd} = d_{ucd} = 8$.

We consider it appropiate to study the large-m limit of this model by truncating the GLW free energy to cubic order. The reason is the following: the EA m-component spin glass model has also been studied, by including up to the Q^4 terms in the GLW free energy (Pytte and Rudnick 1978), for finite m. Although quartic terms should be negligible close to the critical temperature, they play an important role in the finite-m case, since one of them destabilizes the theory by inducing a negative gap in one of the correlation functions. However, in the large $m \to \infty$ limit, this instability is removed (Bray and Moore 1978). As a counterpart, the Q^3 model presents marginally stable solutions which show up as gapless correlation functions within mean-field theory, for any value of m.

2. The model

We consider the m-component Edwards–Anderson spin-glass model, in the limit $m \to \infty$ independently of the spatial dimensionality d. In this model, spins are located at the sites of an hypercubic d-dimensional lattice, and each of them interacts with its z nearest neighbours via quenched random exchange coefficients $\{J_{ij}\}$, whose values obey a Gaussian probability density $P(J_{ij})$. In order to obtain the free energy F, we need to calculate the average of $\ln Z\{J_{ij}\}$ over the randomness in the exchange coefficients, $Z\{J_{ij}\}$ being the partition function for a particular realization of bonds $\{J_{ij}\}$. To this end, we make use of the replica method (Edwards and Anderson 1975), which consists of the use of the relation $\langle \ln Z\rangle = \lim_{n\to 0}[\langle Z^n\rangle - 1]/n$, where Z^n is the partition function for n identical copies, or replicas, of the original system; therefore, the quantity to be calculated is $\lim_{n\to 0}\langle Z^n\rangle_J$, where $\langle\ \rangle_J$ indicates average over randomness in the exchange coefficients.

The correctness of the use of the replica method in the study of spin glasses was highly debated in the past (Anderson 1983, Almeida and Thouless 1978), because of the 'unphysical' results obtained at low temperatures, until Bray and Moore (1978) suggested that the existence of modes with a negative gap implied that the symmetry between replicas should be broken in order to represent the physics of the problem. It is now believed that, for any finite m, the value of the order parameter $q_{\alpha\beta}$ relating two different replicas (α, β), should depend on the specific replicas involved, in a way similar to a hierarchical tree with ultrametric structure (Mèzard *et al* 1984). In this way, different possible evolutions of the system are reflected on the different values of the order parameters. Mathematically, this corresponds to an infinite succesion of symmetry breaking between replicas as temperature is lowered, where $q_{\alpha\beta} \to q(x)$ becomes a continuous variable (Parisi 1979, 1980a,b,c). However, for infinite m the replica method is expected to give an exact result without the need to break symmetry, since the model recovers ergodicity in this limit. The former statement is supported by numerical evidence (Morris *et al* 1986), and by previous studies which show that the replica-symmetric method gives an exact solution in the $m \to \infty$ limit of the long-range vector model (Kosterlitz *et al* 1976, de Almeida *et al* 1978), together with the knowledge that instabilities of the Q^4 EA SG model are removed in this large-m limit (Bray and Moore 1978).

By following the approach used by Bray and Moore (1979a), we start with the usual model of a spin-glass Hamiltonian and consider the partition function of the system replicated n times,

$$\langle Z^n \rangle_J = \int \prod_{(i,j)} \mathrm{d}J_{ij}\, P(J_{ij})\, \mathrm{Tr}\Big(\exp\Big[(\tfrac{1}{2}\beta) \sum_{\substack{ij \\ \alpha\mu}} J_{ij} S^\alpha_{i\mu} S^\alpha_{j\mu}\Big]\Big)$$

where $S^\alpha_{i\mu}$ is the μth Cartesian component of the spin located at the site i and belonging to the replica α and the trace has to be taken over all spin configurations. After averaging over the exchange coefficients, we obtain

$$\langle Z^n \rangle_J = \mathrm{Tr}\Big[\exp\Big(\frac{1}{2} \sum_{ij} K_{ij} \sum_{\alpha\beta\mu\nu} S^\alpha_{i\mu} S^\alpha_{j\mu} S^\beta_{i\nu} S^\beta_{j\nu}\Big)\Big]$$

where $K_{ij} = \frac{1}{2}(\Delta/T)^2$ for i, j nearest neighbours and zero otherwise, Δ is the width of the Gaussian bond distribution, and T is the temperature. We now introduce a set of auxiliary fields $\{Q^{\alpha\beta}_{\mu\nu}\}$ in order to decouple the lattice sites via a Hubbard Stratonovich transformation (Hubbard 1959) to obtain

$$\langle Z^n \rangle_J = \int \prod_{\substack{i\alpha\beta \\ \mu\nu}} \mathrm{d}Q^{\alpha\beta}_{i\mu\nu} \exp\Big(-\frac{1}{2} \sum_{\substack{ij\alpha \\ \beta\mu\nu}} K^{-1}_{ij} Q^{\alpha\beta}_{i\mu\nu} Q^{\alpha\beta}_{j\mu\nu} + \ln\Big[\mathrm{Tr}_\alpha \exp\Big(\sum Q^{\alpha\beta}_{i\mu\nu} S^\alpha_{i\mu} S^\beta_{i\nu}\Big)\Big]\Big). \tag{1}$$

These fields act as order parameters of the theory and for convenience they can be separated into two parts, each having different physical meaning:

$$Q^{\alpha\beta}_{\mu\nu} \to Q^{\alpha\beta}_{\mu\nu}(1 - \delta^{\alpha\beta}) + \delta^{\alpha\beta}[Q^{\alpha\alpha}\delta_{\mu\nu} + T^\alpha_{\mu\nu}]. \tag{2}$$

The non-diagonal part of $Q^{\alpha\beta}_{\mu\nu}$ with $\alpha \neq \beta$ accounts for the interaction between different replicas α, β, and therefore it represents the spin-glass order. On the other hand, the term within the squared parenthesis represents the interaction between spin components of the same replica (α), and its trace in the spin space $\sum_\mu [Q^{\alpha\alpha}\delta_{\mu\mu} + T^\alpha_{\mu\mu}]$ represents a 'hard' non-critical mode. The fields $Q^{\alpha\alpha}$ decouple from other fields and can be integrated out of the problem; as a consequence, the remaining part $T^\alpha_{\mu\nu}$ is a traceless tensor usually called 'quadrupole field'. After introducing (2) into (1), expanding the external exponent to third order in the spin-glass $Q^{\alpha\beta}_{\mu\nu}$ and quadrupole $T^\alpha_{\mu\nu}$ fields, carrying out the spin traces, re-exponentiating and taking the continuum limit, we obtain to lowest order in the derivatives

$$\langle Z^n \rangle_J = \int \prod_{\substack{\alpha\beta\gamma \\ \mu\nu\rho\tau}} \mathrm{d}Q^{\alpha\beta}_{\mu\nu}(x)\, \mathrm{d}T^\gamma_{\rho\tau}(x) \exp\Big(-\int \mathcal{F}\{Q^{\alpha\beta}_{\mu\nu}(x), T^\gamma_{\rho\tau}(x)\}\Big) \tag{3a}$$

with

$$\mathcal{F} = \frac{1}{4}\sum\left[(Q_{\mu\nu}^{\alpha\beta})^2 + r(Q_{\mu\nu}^{\alpha\beta})^2\right] + \frac{1}{4}\sum\left[(T_{\mu\nu}^{\alpha})^2 + (r + \frac{\tau}{m+2})(T_{\mu\nu}^{\alpha})^2\right]$$
$$- \frac{wm^2}{6(m+2)(m+4)}\sum T_{\mu\nu}^{\alpha} T_{\nu\rho}^{\alpha} T_{\rho\mu}^{\alpha} - \frac{w}{6}\sum Q_{\mu\nu}^{\alpha\beta} Q_{\nu\rho}^{\beta\gamma} Q_{\rho\mu}^{\gamma\alpha}$$
$$- \frac{wm}{2!(m+2)}\sum T_{\mu\nu}^{\alpha} Q_{\mu\rho}^{\alpha\beta} Q_{\rho\nu}^{\alpha\beta} \tag{3b}$$

where the summations run over all free indices. In this expression τ and w are positive functions and the restrictions $Q_{\mu\nu}^{\alpha\alpha} = 0$ and $\sum_{\mu} T_{\mu\mu}^{\alpha} = 0$ apply. We can appreciate from equation (3) that $Q_{\mu\nu}^{\alpha\beta}$ and $T_{\mu\nu}^{\alpha}$ have a mass difference of order $(1/m)$, which disappears as $m \to \infty$, thus changing the critical properties of the model in this limit.

3. Mean-field theory

The replica symmetric mean-field solution of this model, close to the critical temperature T_c, is given by (LV):

$$Q_{\mu\nu}^{\alpha\beta} = \begin{cases} 0 & \text{for } r > 0 \\ r/[w(n-2)]\delta_{\mu\nu} & \text{for } r < 0 \end{cases}$$
$$T_{\mu\nu}^{\alpha} = 0 \qquad \text{for any } r.$$

Therefore $r = 0$ defines T_c. If we expand the GLW free energy density around the mean-field solution in the low-temperature region, and make a Fourier transform of the fields into the momentum space q, we can write

$$\int \mathrm{d}^d x[-\mathcal{F}\{Q(x), T(x)\}] \to \sum_q [-\mathcal{F}_0(q) + \mathcal{F}_1(q) + \mathcal{F}_2(q)] \tag{4}$$

with

$$\mathcal{F}_0 = \frac{1}{4}(q^2 - r)\sum(R_{\mu\nu}^{\alpha\beta})^2 + \frac{1}{4}(q^2 + \frac{\tau}{m+2} - r)\sum(S_{\mu\nu}^{\alpha})^2 \tag{5a}$$

$$\mathcal{F}_1 = \frac{r}{6(n-2)}\sum(R_{\mu\nu}^{\alpha\beta} R_{\nu\mu}^{\beta\gamma} + R_{\mu\nu}^{\alpha\beta} R_{\mu\nu}^{\alpha\gamma} + R_{\mu\nu}^{\alpha\gamma} R_{\mu\nu}^{\beta\gamma})$$
$$+ \frac{rm}{2(m+2)(n-2)}\sum S_{\mu\nu}^{\alpha} R_{\mu\nu}^{\alpha\beta} \tag{5b}$$

$$\mathcal{F}_2 = \frac{w}{6}\sum R_{\mu\nu}^{\alpha\beta} R_{\nu\rho}^{\beta\gamma} R_{\rho\mu}^{\gamma\alpha} + \frac{wm^2}{6(m+2)(m+4)}\sum S_{\mu\nu}^{\alpha} S_{\nu\rho}^{\alpha} S_{\rho\nu}^{\alpha}$$
$$+ \frac{wm}{2(m+2)}\sum S_{\mu\nu}^{\alpha} R_{\mu\rho}^{\alpha\beta} R_{\nu\rho}^{\alpha\beta} \tag{5c}$$

where $R_{\mu\nu}^{\alpha\beta}$ and $S_{\mu\nu}^{\alpha}$ are fluctuations of the spin-glass and quadrupole fields, respectively, about the mean-field solution. There are 23 different propagators or correlation functions between pairs of fluctuations, these are given by:

$$\langle\langle\quad\rangle\rangle_2 = \frac{\int \mathrm{d}R\,\mathrm{d}S(\quad)\exp[-\mathcal{F}\{R, S\}]}{\int \mathrm{d}R\,\mathrm{d}S\exp[-\mathcal{F}\{R, S\}]} \tag{6}$$

where the empty parentheses ( ) represent the quantity to be calculated, $\mathcal{F}$ is the GLW free energy density given by equations (4)–(5), and the subindex 2 indicates that this correlation includes terms up to those included in $\mathcal{F}_2$. In order to obtain the value of these propagators, it is possible to calculate them exactly to quadratic order in R and S, and then to introduce the cubic contribution as a perturbation, which in principle can be calculated to any desired order. The (exact) calculation to quadratic order can be done in the following way: (1) $\mathcal{F}_0$ is used as the free energy functional and the three possible bare propagators are calculated; these are: $\langle R^{\alpha\beta}_{\mu\nu}(q)R^{\alpha\beta}_{\mu\nu}(-q)\rangle_0$, $\langle S^{\alpha}_{\mu\rho}(q)S^{\alpha}_{\mu\rho}(-q)\rangle_0$ and $\langle S^{\alpha}_{\mu\mu}(q)S^{\alpha}_{\nu\nu}(-q)\rangle_0$, with $\mu \neq \rho$, $\alpha \neq \beta$; (2) $\mathcal{F}_0 - \mathcal{F}_1$ is used as the energy density functional in equation (6) and $\exp\{\mathcal{F}_1\}$ is expanded. The resulting series is summed to all orders in terms of the bare propagators. In this way, a set of 23 Dyson-type equations—which has to be solved—is obtained. Details of this calculation have been given elsewhere (Viana 1985).

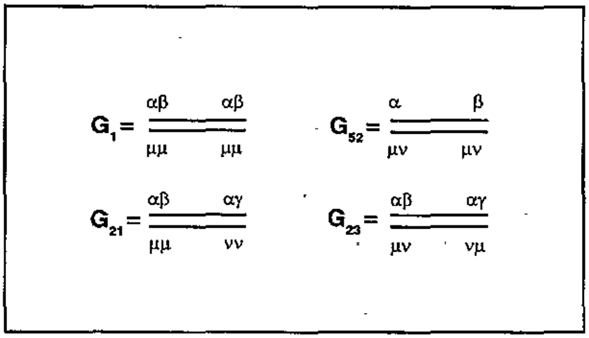

Figure 1. This figure shows the graphical notation for three typical propagators. Here, upper and lower indices are the replica and component indices respectively, with $\alpha \neq \beta \neq \gamma \neq \delta$ and $\mu \neq \nu$; double lines indicate these are 'dressed' propagators, given exactly to quadratic order.

An example of the diagramatic representation of the correlation functions calculated to quadratic order is shown in figure 1. These are represented by a double line, the greek symbols located at the top of this line indicate the replica indices ($\alpha \neq \beta \neq \gamma$), and the lower indices indicate the spin components ($\mu \neq \nu$); finally, each side of the graph corresponds to each of the two fluctuations involved. The mathematical notation for these correlation functions is given by:

$$
\begin{aligned}
G_1 &= \langle R^{\alpha\beta}_{\mu\mu}(q)R^{\alpha\beta}_{\mu\mu}(-q)\rangle_1 & G_{11} &= \langle R^{\alpha\beta}_{\mu\mu}(q)R^{\alpha\beta}_{\nu\nu}(-q)\rangle_1 \\
G_2 &= \langle R^{\alpha\beta}_{\mu\mu}(q)R^{\alpha\gamma}_{\mu\mu}(-q)\rangle_1 & G_{21} &= \langle R^{\alpha\beta}_{\mu\mu}(q)R^{\alpha\gamma}_{\nu\nu}(-q)\rangle_1 \\
G_3 &= \langle R^{\alpha\beta}_{\mu\mu}(q)R^{\gamma\delta}_{\mu\mu}(-q)\rangle_1 & G_{31} &= \langle R^{\alpha\beta}_{\mu\mu}(q)R^{\gamma\delta}_{\nu\nu}(-q)\rangle_1 \\
G_4 &= \langle S^{\alpha}_{\mu\mu}(q)S^{\alpha}_{\mu\mu}(-q)\rangle_1 & G_{41} &= \langle S^{\alpha}_{\mu\mu}(q)S^{\alpha}_{\nu\nu}(-q)\rangle_1 \\
G_5 &= \langle S^{\alpha}_{\mu\mu}(q)S^{\beta}_{\mu\mu}(-q)\rangle_1 & G_{51} &= \langle S^{\alpha}_{\mu\mu}(q)S^{\beta}_{\nu\nu}(-q)\rangle_1 \\
G_6 &= \langle R^{\alpha\beta}_{\mu\mu}(q)S^{\alpha}_{\mu\mu}(-q)\rangle_1 & G_{61} &= \langle R^{\alpha\beta}_{\mu\mu}(q)S^{\alpha}_{\nu\nu}(-q)\rangle_1 \\
G_7 &= \langle R^{\alpha\beta}_{\mu\mu}(q)S^{\gamma}_{\mu\mu}(-q)\rangle_1 & G_{71} &= \langle R^{\alpha\beta}_{\mu\mu}(q)S^{\gamma}_{\nu\nu}(-q)\rangle_1 \\
G_{12} &= \langle R^{\alpha\beta}_{\mu\nu}(q)R^{\alpha\beta}_{\mu\nu}(-q)\rangle_1 & G_{22} &= \langle R^{\alpha\beta}_{\mu\nu}(q)R^{\alpha\gamma}_{\mu\nu}(-q)\rangle_1 \\
G_{32} &= \langle R^{\alpha\beta}_{\mu\nu}(q)R^{\gamma\delta}_{\mu\nu}(-q)\rangle_1 & G_{42} &= \langle S^{\alpha}_{\mu\nu}(q)S^{\alpha}_{\mu\nu}(-q)\rangle_1 \\
G_{52} &= \langle S^{\alpha}_{\mu\nu}(q)S^{\beta}_{\mu\nu}(-q)\rangle_1 & G_{62} &= \langle R^{\alpha\beta}_{\mu\nu}(q)S^{\alpha}_{\mu\nu}(-q)\rangle_1
\end{aligned}
$$

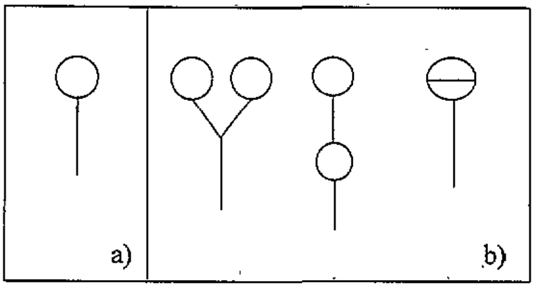

Figure 2. Squeleton graphs for the first- and second-order terms in the perturbation expansion of $\langle R^{\alpha\beta}_{\mu\mu}(0)\rangle_2$, to include the cubic contribution to $\mathcal{F}$. Each of the points at which three lines meet is proportional to w.

$$G_{72} = \langle R^{\alpha\beta}_{\mu\nu}(q) S^{\gamma}_{\mu\nu}(-q)\rangle_1 \qquad G_{13} = \langle R^{\alpha\beta}_{\mu\nu}(q) R^{\alpha\beta}_{\nu\mu}(-q)\rangle_1$$
$$G_{23} = \langle R^{\alpha\beta}_{\mu\nu}(q) R^{\alpha\gamma}_{\nu\mu}(-q)\rangle_1 .$$

The mathematical value of these propagators, or of simple combinations of them, can be found elsewhere (LV, Viana 1985). However, it is important to point out that in the limit $m \to \infty$, all 'dressed' pure propagators are massless, as any propagation in a direction within the subspace generated by the eigenvectors corresponding to massless modes, can be done at no energy cost. There are some simple combinations of propagators which have simple poles and appear naturally in the theory. Among them, we have the following exact expressions (valid for any value of m):

$$\mathcal{G}_B = \mathcal{G}_1 - 4\mathcal{G}_2 + 3\mathcal{G}_3 = \frac{1}{(q^2 + |r|)}$$
$$\mathcal{G}_R = \mathcal{G}_1 - 2\mathcal{G}_2 + \mathcal{G}_3 = \frac{1}{q^2}$$

where we have defined $\mathcal{G}_i = G_i + (m-1)G_{i1}$. These modes are the infinite component analogues of the 'breathing' and 'replicon' modes for the finite-m case (Bray and Moore 1979a,b), as they have the same mathematical expression.

4. Perturbation theory

According to equation (6), the shift in the order parameter $\langle R^{\alpha\beta}_{\mu\mu}\rangle$ introduced by the presence of the cubic term in $\mathcal{F}$, given by $\mathcal{F}_2$ (equation(5c)), can be written as the sum of all connected diagrams of the type

$$\langle R^{\alpha\beta}_{\mu\nu}\rangle_2 = \sum_{l=0}^{\infty} \frac{1}{l!} \langle R^{\alpha\beta}_{\mu\nu}(\mathcal{F}_2)^l\rangle_1 \tag{7}$$

and calculated, in principle, to any desired order. Figure 2 shows the 'squeleton' (unlabeled) graphs for the first (a) and second (b) non-trivial orders in the series. In order to evaluate the contribution of these 'squeleton' graphs it is necesary to sum over all possible graphs that 'fit' into the structure indicated by the squeletons, that is, over all connected diagrams having the same number of (double) lines, each of them representing the correlation between two fluctuations involved. Each of the points at which any three lines meet is proportional to w and $(np+1)/2$ gives the non-trivial order of the term, where np is the number of such points.

The first non-trivial order ($l = 0$) of the expression (7), known as one-loop approximation or tadpole graph $\langle R^{\alpha\beta}_{\mu\nu}\rangle_1$, has been calculated (LV), with the following result:

$$\langle R^{\alpha\beta}_{\mu\mu}(q=0)\rangle_1 = \frac{w}{|r|}\sum_q\left(\frac{1}{q^8}\left[q^4(-3|r|/2)+q^2(-|r|^2-|r|\tau/2)+(3|r|^3-3|r|^2\tau/4)\right]\right) + \mathrm{O}(1/m). \tag{8}$$

In the infinite volume limit, it is permissible to replace the sum in equation (8) by an integral in the d-dimensional space $\Sigma_q \to \int L^d \mathrm{d}q^d/(2\Pi)^d$. Thus, the first non-trivial perturbative correction diverges for $d \leqslant 8$, which could be an indication that $d = 8$ is the lower critical dimension of the theory. On the other hand, by analysing the behaviour of this model in the high-temperature phase, Green *et al* (1982) found to the same order in perturbation theory that the upper critical dimension of the theory is eight. In order to study this bizarre behaviour, we went to higher order in the expansion given by equation (7). It could be expected that each time a diagram presents a tadpole segment, a term proportional to q^{-8} would appear. However, we evaluated two graphs of the next non-trivial order to see if there were additional divergences. We found the following results: to order $\mathrm{O}(1/m)$, the contribution of the first graph shown in figure 2(b) is given by

$$-\left(\frac{w}{|r|}\right)^3\left(\sum_q\frac{1}{q^8}\left[q^4(-\tfrac{3}{2}|r|)+q^2(-\tfrac{1}{2}|r|\tau-|r|^2)+(3|r|^3-\tfrac{3}{4}|r|^2\tau)\right]\right)^2 \tag{9a}$$

which also diverges for for $d \leqslant 8$. However, if we consider the second graph on figure 2(b), we can see that this is given by

$$\left(\frac{w^3}{4|r|^2}\right)\left(\sum_q\frac{1}{q^8}\left[q^4(-\tfrac{3}{2}|r|)+q^2(-\tfrac{1}{2}|r|\tau-|r|^2)+(3|r|^3-\tfrac{3}{4}|r|^2\tau)\right]\right)$$
$$\times\left(\sum_q\frac{1}{q^{14}}\left[q^{10}(-6)+q^8(-2\tau+4|r|)+q^6(48|r|^2)\right.\right.$$
$$\left.\left.+q^4(12|r|^2\tau-24|r|^3)+q^2(12|r|^3\tau+8|r|^4)+(32|r|^4\tau-128|r|^5)\right]\right) \tag{9b}$$

whose value diverges for any dimension $d \leqslant 14$. This result suggests to us that the dimensionality, below which the terms of the perturbation expansion diverge, grows with the order of the terms. Therefore, it is necessary to make a deeper analysis before attempting to draw any conclusion on the value of the lower critical dimensionality.

5. Discussion

As we have just mentioned, in the macroscopic limit every summation can be replaced by an integration over q. However, each integral is IR divergent below certain dimension D_{IR} which can be identified with the lower critical dimension d_{lcd} of the theory. It is thus necessary to introduce some regularization scheme in order to turn each IR-divergent integral into a well defined object. The divergencies are related to the fact that in the $m \to \infty$ limit, zero-mass loops appear in each diagram. In fact, the dimension for which the theory diverges rises with the order of the perturbation.

An approach well suited to handle this behaviour is the dimensional regularization scheme. Within this scheme, the IR divergence shows up as a pole when the space dimensionality is D_{IR} ($q \to 0$); however, there also exists a pole arising from the ultraviolet (UV) divergence for a definite dimension D_{UV} ($q \to \infty$), which is found to cancel exactly with the IR divergence. The dimensional regularization can be performed by introducing a cut-off parameter Λ which allows us to make an analytic continuation of the integral to arbitrary values of D_{IR} and D_{UV}. It turns out that by choosing $D_{IR} = D_{UV}$, the pole arising from the UV divergence cancels exactly with the pole corresponding to the IR divergence, so the integral is null—in the dimensional regularization sense—for any arbitrary dimension (see appendix). By following the same procedure at every order in perturbation theory, we find that all terms are null in the dimensional regularization sense.

In general, the dimensionality D_{UV} is not relevant in the discussion of critical phenomena; however, the fact that in this problem we can choose $D_{IR} = D_{UV}$, is related to the finding that there is no characteristic scale in this problem; this lack of a characteristic scale is also reflected in the absence of correction terms to Q proportional to $\log(T - T_c)$.

The previous results, in the low-temperature region, can be interpreted in two different ways. First, they could be an indication that replicons destroy long-range order with no energy loss; this would mean that any small perturbation would change instantly the magnetization in large areas. Another interpretation, which we think is more adequate, is that the role of these infinite-range fluctuations would be constrained to transport information between replicas at no energy cost. This would mean, in turn, that in the $m \to \infty$ limit the free energy valleys in configuration space are connected through zero-altitude passages, in agreement with the idea that in this limit the ergodicity of the system is restored.

Appendix

Consider the integral

$$I = \int \frac{\mathrm{d}^D q}{(q^2)^\alpha} = \frac{\Pi^{D/2}}{\Gamma(D/2)} \int_0^\infty (q^2)^{D/2-\alpha-1}\, \mathrm{d}(q^2) \tag{A.1}$$

where the right-hand side is the expression in D-dimensional polar coordinates. The integral is UV divergent for $D > 2\alpha$ and IR divergent for $D < 2\alpha$. Let us split the integration into an UV part, $q^2 > \Lambda^2$ and an infrared part $q^2 < \Lambda^2$:

$$I = \frac{\Pi^{d/2}}{\Gamma(D/2)} \int_0^{\Lambda^2} \mathrm{d}q^2 (q^2)^{D/2-\alpha-1} + \int_{\Lambda^2}^\infty \mathrm{d}q^2 (q^2)^{D/2-\alpha-1}. \tag{A.2}$$

The first integral is convergent for $D = D_{IR} > 2\alpha$, and the second one for $D = D_{UV} < 2\alpha$. By performing the integrations for $D_{IR} > 2\alpha$ and $D_{UV} < 2\alpha$, we get

$$I = \frac{\Pi^{D/2}}{\Gamma(D/2)} \left(\frac{\Lambda^{D_{IR}-2\alpha}}{\frac{1}{2}D_{IR} - \alpha} - \frac{\Lambda^{D_{UV}-2\alpha}}{\frac{1}{2}D_{UV} - \alpha} \right). \tag{A.3}$$

The two terms show poles for $D_{UV} = D_{IR} = 2\alpha$. However, this expression can be continued analytically to arbitrary values of D_{IR} and D_{UV}, and the constraints $D_{IR} > 2\alpha$ and $D_{UV} < 2\alpha$ can be removed. By identifying $D_{UV} = D_{IR}$, the integral vanishes (Muta 1987, Le Bellac 1991).

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